SELECTION OF MIXED SAMPLING PLAN WITH SINGLE SAMPLING PLAN AS ATTRIBUTE PLAN INDEXED THROUGH (MAPD, MAAOQ) AND (MAPD, AOQL)

R. Sampath Kumar*

R. Kiruthika*

R. Radhakrishnan*

ABSTRACT:

This paper presents the procedure for the construction and selection of the mixed sampling plan using MAPD as a quality standard and MAAOQ as a measure for outgoing quality with the single sampling plan (SSP) as attribute plan using weighted Poisson distribution as a base line distribution. The plans indexed through (MAPD, MAAOQ) and (MAPD, AOQL) are constructed and compared. Tables are provided for the easy selection of the plans.

Keywords and Phrases: Maximum allowable percent defective, Maximum allowable average outgoing quality level, Average outgoing quality level, Operating Characteristic.

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* Assistant Professor, Department of Statistics, Government Arts College, Coimbatore.

* Assistant Professor in Statistics, Kalaignar Karunanidhi Institute of Technology, Coimbatore.

* Associate Professor, Department of Statistics, PSG College of Arts and Science, Coimbatore.

1. INTRODUCTION:

Mixed sampling plans consist of two stages of rather different nature. During the first stage the given lot is considered as a sample from the respective production process and a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted. Otherwise the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, then the plan is said to be independent otherwise dependent. The principal advantage of a mixed sampling plan over pure attribute sampling plans is a reduction in sample size for a similar amount of protection.

The concept of MAPD (p_*) was introduced by Mayer (1967) and further studied by Soundararajan (1975) is the quality level corresponding to the inflection point on the OC curve. The degree of sharpness of inspection about this quality level ‘ p_* ’ is measured through ‘ p_t ’, the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis.

The mixed sampling plan has been designed under two cases of significant interest. In the first case sample size n_1 is fixed and a point on the OC curve is given. In the second case plans are designed when two points on the OC curve are given. The procedure for designing the mixed sampling plans to satisfy the above mentioned conditions was provided by Schilling (1967). Using Schilling’s procedure, Devaarul (2003) has constructed tables for mixed sampling plans (independent case) having various sampling plans as attribute plans. Radhakrishnan and Sampath Kumar (2006, 2007) and Radhakrishnan et.al (2010) have made contributions to mixed sampling plan for independent case.

Sampath Kumar (2007) has constructed mixed sampling plan with various sampling plans as attribute plans using Poisson distribution as a base line distribution. Radhakrishna Rao (1977) suggested a weighted Binomial distribution can be used in designing sampling plans. Sudeswari (2002) studied the construction of sampling plans using weighted Poisson distribution as a base line distribution. Mohana Priya (2008), Radhakrishnan and Mohana Priya (2008 a, 2008 b) have constructed the sampling plans using weighted Poisson distribution as a base line distribution.

The weighted Poisson distribution plays an important role in the acceptance sampling, mainly in the construction of sampling plans. Each outcome (number of defectives) is specific but can be assigned with different weights based on its importance or usage.

In this paper, the mixed sampling plan (independent case) with single sampling plan as attribute plan is constructed using weighted Poisson distribution as a base line distribution. Tables are constructed for the mixed sampling plan indexed through i) MAPD, MAAOQ (ii) MAPD, AOQL. The plan indexed through (MAPD, MAAOQ) is also compared with the plan indexed through (MAPD, AOQL).

2. GLOSSARY OF SYMBOLS:

The symbols used in this paper are as follows:

p : submitted quality of lot or process

$P_a(p)$: probability of acceptance for given quality p

p_* : maximum allowable percent defective

p_m : the product quality at which AOQ is maximum

p_t : the point at which the inflection tangent of the OC curve cuts the 'p' axis

h_* : relative slope at p_*

n_1 : sample size for the variable sampling plan

n_2 : sample size for the attribute sampling plan

c : acceptance number

β_j : probability of acceptance for lot quality 'p_j'

β_j' : probability of acceptance assigned to first stage for percent defective p_j

β_j'' : probability of acceptance assigned to second stage for percent defective p_j

k : variable factor such that a lot is accepted if $\overline{X} \leq A = U - k\sigma$

3. OPERATING PROCEDURE OF MIXED SAMPLING PLAN HAVING SINGLE SAMPLING PLAN AS ATTRIBUTE PLAN:

Schilling (1967) has given the following procedure for the independent mixed sampling plan with upper specification limit (U) and standard deviation (σ).

1. Determine the parameters of the mixed sampling plan n_1, n_2, k and c.
2. Take a random sample of size n_1 from the lot.
3. If the sample average $\overline{X} \leq A = U - k\sigma$, accept the lot
4. If the sample average $\overline{X} > A = U - k\sigma$, take a second sample of size n_2 and count the number of defectives'd' in the sample.
   (i) If $d \leq c$, accept the lot.
   (ii) If $d > c$, reject the lot.

4. CONSTRUCTION OF MIXED SAMPLING PLAN HAVING SINGLE SAMPLING PLAN AS ATTRIBUTE PLAN USING WEIGHTED POISSON DISTRIBUTION:

The detailed procedure adopted in this paper for the construction of mixed sampling plan having single sampling as attribute plan using weighted Poisson distribution indexed through MAPD is given below:

- Assume that the mixed plan is independent
- Decide the sample size n_1 (for variable sampling plan) to be used.
- Calculate the acceptance limit for the variable sampling plan as

$A = U - [z(p_*) + \{z(\beta_*^{'}) / \sqrt{n_1}\}]\ \sigma$, where z is standard normal variate corresponding to't'

such that t = $\int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$

- Split the probability of acceptance β_* as β_*' and β_*'' such that $\beta_* = \beta_*' + (1- \beta_*')\ \beta_*''$. Fix the value of β_*'
- Determine β_*'', the probability of acceptance assigned to the attribute plan associated with the second stage sample as $\beta_*'' = (\beta_* - \beta_*')/ (1- \beta_*')$.
- Determine the appropriate second stage sample of n_2 from the relation

$$\beta_*'' = \sum_{r=1}^{c} \frac{e^{-n_2 p}(n_2 p)^{r-1}}{(r-1)!}$$

Using the above procedure, tables have been constructed to facilitate easy selection of mixed sampling plan using single sampling plan as attribute plans indexed through MAPD.

4.1 Construction of tables

The probability of acceptance for single sampling plan under weighted Poisson distribution is $P_a(p) = \frac{x^\alpha p(x,\alpha)}{\sum_{x=0}^{\infty} x^\alpha p(x;\alpha)}$; x=0, 1, 2… where $p(x;\alpha) = \frac{e^{-np}(np)^x}{x!}$; x=0, 1, 2….

The probability of acceptance for single sampling plan under weighted Poisson distribution when α = 1 is used in this paper for determining the second stage probabilities and is given by

$$P_a(p) = \frac{e^{-n_2 p}(n_2 p)^{x-1}}{(x-1)!}, \quad x=1, 2, 3\ldots..$$

The inflection point (p_*) is obtained by using $\frac{d^2 p_a(p)}{dp^2} = 0$ and $\frac{d^3 p_a(p)}{dp^3} \neq 0$.

Maximum Allowable Average Outgoing Quality (MAAOQ) is the average outgoing quality at the inflection point. This section provides the procedure of constructing the mixed sampling plans having single sampling plan as attribute plan indexed through (MAPD, MAAOQ) and (MAPD, AOQL). The average outgoing quality (AOQ) function is given by

$$AOQ = p.\ Pa\ (p)$$

$$MAAOQ = p_*.\ Pa\ (p_*)$$

$$\text{and } AOQL = p_m.\ Pa\ (p_m)$$

The values of β_*'', n_2p_*, n_2MAAOQ, n_2AOQL are calculated for different possible combinations of 'c' for $\beta_*' = 0.30$ using C program and presented in Table 1.

4.1 Selection of the plan

Table 1 is used to construct the plans for a specified MAAOQ and MAPD (p_*). For any given values of p_* and MAAOQ one can find the ratio R_1=MAAOQ/p_*. Using Table 1, select the nearest value of R_1, the corresponding 'c' value is noted. The n_2 value is determined using $n_2=n_2p_*/p_*$.

Table 1 is used to construct the plans for a specified AOQL and MAPD (p_*). For any given values of p_* and AOQL one can find the ratio R_2=AOQL/p_*. Using Table 1, select the nearest value of R_2, the corresponding 'c' value is noted. The n_2 value is determined using $n_2=n_2p_*/p_*$.

Example 1: Given p_*=0.07, MAAOQ = 0.03 and β_*' = 0.30. Find the ratio R_1=MAAOQ/p_* = 0.4286 and select the nearest value of R_1 using Table 1 as 0.4376 which is associated with c= 7 and $n_2=n_2p_*/p_*$=7.0819/0.07=101.Thus n_2=101 and c=7 are the parameters selected for the mixed sampling plan with single sampling plan as attribute plan for a specified p_*=0.07, MAAOQ = 0.03.

Example 2: Given p_*=0.01, AOQL = 0.005 and β_*' = 0.30. Find the ratio R_2 =AOQL/p_* = 0.5000 and select the nearest value of R_2 using Table 1 as 0.5019 which is associated with c = 4 and $n_2=n_2p_*/p_*$=1.8528/0.01=185.Thus n_2=185 and c=4 are the parameters selected for the mixed sampling plan with single sampling plan as attribute plan for a specified p_*=0.01, AOQL = 0.005.

5. COMPARISON OF MIXED SAMPLING PLAN WITH SINGLE SAMPLING PLAN AS ATTRIBUTE PLAN INDEXED THROUGH (MAPD, MAAOQ) AND (MAPD, AOQL):

In this section mixed sampling plan with single sampling plan as attribute plan indexed through (MAPD, MAAOQ) is compared with the mixed sampling plan with single sampling plan as attribute plan indexed through (MAPD, AOQL) by fixing the parameters p_* and MAAOQ = AOQL.

Example 3:

Given MAPD = 0.010, MAAOQ=0.005=AOQL and β_*' = 0.30. Find the ratio $R_1=R_2=0.500$ and select the nearest values of R_1 and R_2 from Table 1 as $R_1=0.5381$ and $R_2=0.5019$ which are associated with c =3 and c=4 respectively. From this one can find $n_2=n_2p_*/p_*=2.5221/0.010$ = 252 for c=3 and p_* =0.010 and $n_2=n_2p_*/p_*=3.6913/0.010=369$ for c=4 and $p_*=0.010$. Thus $n_2=252$ and c=3 are the parameters selected for the mixed sampling plan with single sampling plan as attribute plan for a specified MAPD = 0.010, MAAOQ=0.005 and $n_2=369$, c=4 are the parameters selected for the mixed sampling plan with single sampling plan as attribute plan for a specified MAPD = 0.010, AOQL=0.005.

6. CONSTRUCTION OF AOQ CURVE:

The AOQ curves for the plans $n_2=252$ and c=3 (indexed through MAPD, MAAOQ) and $n_2=369$ and c=4 (indexed through MAPD, AOQL) based on the different values of n_2p and AOQ are presented in Figure 1.

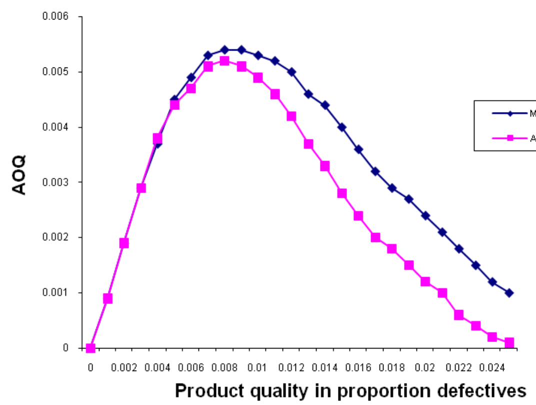

Figure 1: AOQ curves for SSP

7. CONCLUSION:

In this paper the procedure for constructing mixed sampling plans with single sampling plan as attribute plan indexed through (MAPD, MAAOQ) and (MAPD and AOQL) with weighted Poisson distribution as the baseline distribution are presented and compared. Suitable tables are also provided for easy selection of the plans for the engineers who are working on the floor of the assembly. It is concluded from the study that the second sample size required for mixed sampling plan with single sampling plan as attribute plan indexed through (MAPD,MAAOQ) is less than that of the second stage sample size of the mixed sampling plan with single sampling plan as attribute plan indexed through (MAPD,AOQL), justified by Sampath Kumar (2008). These plans definitely help the producers, because of the lesser sample size which directly result in lesser sampling cost and indirectly reduces the total cost of the product.

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Table 1: Certain Parametric Values of Single sampling plan for β_*' =0.30

c	β_*''	n_2p_*	n_2MAAOQ	n_2AOQL	R_1	R_2
1	1	0	0	0	-	-
2	0.6226	1.3118	0.8167	0.7448	0.6226	0.5678
3	0.5381	2.5221	1.3571	1.2819	0.5381	0.5083
4	0.4960	3.6913	1.8309	1.8528	0.4960	0.5019
5	0.4697	4.8357	2.2713	2.2713	0.4697	0.5071
6	0.4514	5.9640	2.6921	2.6921	0.4514	0.5154
7	0.4376	7.0819	3.0990	3.0990	0.4376	0.5246
8	0.4267	8.1899	3.4946	3.4946	0.4267	0.5337
9	0.4179	9.2917	3.8830	3.8830	0.4179	0.5426
10	0.4106	10.3858	4.2644	4.2644	0.4106	0.5489
11	0.4043	11.4774	4.6403	4.6403	0.4043	0.5592
12	0.399	12.5617	5.0121	5.0121	0.3990	0.5669
13	0.3943	13.6447	5.301	5.3801	0.3943	0.5740
14	0.3900	14.7239	5.7423	5.7423	0.3900	0.5808
15	0.3893	15.8008	6.1038	6.1038	0.3863	0.5871
16	0.3830	16.8726	6.4622	6.4622	0.3830	0.5933
17	0.3800	17.9432	6.8184	6.8184	0.3800	0.5991
18	0.3771	19.0148	7.1705	7.1705	0.3771	0.6044
19	0.3746	20.0813	7.5225	7.5225	0.3746	0.6096
20	0.3723	21.1448	7.8704	7.8704	0.3722	0.6146